5. Integer and Floating Point Arithmetic
ENEE 140

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Today's Lecture

- Where we've been
  - Using variables and constants
  - Variable assignment and operators
  - Iterating (`while`, `for`) and branching (`if`)
  - `printf()` and `scanf()`
  - Functions
  - Basics of data types
- Where we're going today
  - Unsigned data type
  - The % operator
  - Prefix and postfix increment operators
  - Assignment operators (+=)
  - Integer & floating point arithmetic
  - Overflow and underflow
- Where we're going next
  - Data types and type conversion

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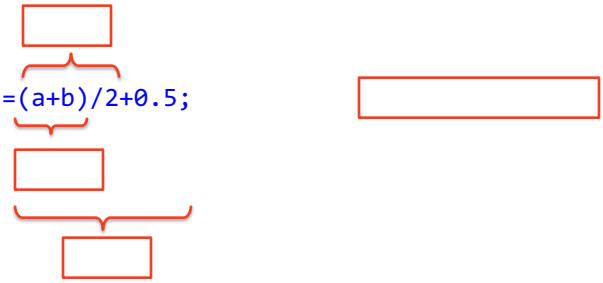
Review: Integer vs. Floating-Point Arithmetic

- What is the value of c?

```
float a = 1;
float b = 2;
int c=(a+b)/2+0.5;
```



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Review: Integer vs. Floating-Point Arithmetic – cont'd

- What is the value of g?

```
int a = 1;
int b = 2;
float c = 1;
float d = 2;
float e = c/d;
int f = a/d;
int g;

if (e > (float)f) {
  g = 1;
} else {
  g = 0;
}
```

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Prefix and Postfix Increment Operators

- We've seen: increment and decrement operators in C

```
a++;        same as a = a + 1;
++a;        same as a = a + 1;
a--;        same as a = a - 1;
--a;        same as a = a - 1;
```

- We've also seen: value of assignment expressions

```
c = b = a = 0;        a, b and c become 0
```

- Both a++ and ++a increment a, but they return different values

```
b = a++;        postfix increment: a becomes 1, b becomes 0
b = ++a;        prefix increment: a becomes 1, b becomes 1
```

  – Same for a-- and --a

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Assignment Operators

- We've seen

```
a++;                increment a by 1
a = a + 10;         increment a by 10
```

- Prefix and postfix operators

```
a = 1;
b = a++;            a becomes 2, b becomes 1
b = ++a;            a becomes 3, b becomes 3
```

- Other assignment operators

```
a += 10;            increment a by 10
```

Same for -=, *=, /=, %=

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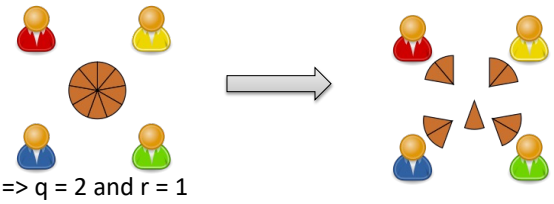
Integer (Euclidean) Division – In Math

- Dividing two integers produces a **quotient (q)** and a **remainder (r)**
  - The quotient and the remainder always exist and are unique
  - Example: dividing a pie with 9 slices among 4 people (source: Wikipedia)



  - 9 / 4 => q = 2 and r = 1
- Mathematical definition:
  - Given integers A and B, with B ≠ 0: there exist **unique** integers r,q such that:
    - A = B*q + r and
    - 0 ≤ r< B

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Integer (Euclidean) Division - Examples

- What are the remainders and quotients when dividing:
  - 8 by 4
  - 4 by 8
  - 10 by 10
- What is the remainder when dividing:
  - $(2^n - 1)$ by 2
  - 2*n by n
  - $(2^n - 1)$ by 2^n

(assume that n is a positive integer)

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Computer Arithmetic – Operations

- We've seen
  `+ - / *` arithmetic operators
  - Work for both integer and floating-point variables
  - Integer division truncates toward 0 (i.e. the fractional part is discarded)

- The modulus operator %
  - Works only for integers
  - Produces the remainder from integer division

```
int a = 5 / 3;
int b = 5 % 3;
```

value of a is

value of b is

- The values of `a % n` range between

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Order of Evaluation

- Operator precedence (complete rules in K&R Table 2.1)
  1. `! ++ --` (unary operators)
  2. `* / %`
  3. `+ -`
  4. `< <= > >=`
  5. `== !=`
  6. `&&`
  7. `||`
  8. `=`

- Rule of thumb:
  - Division and multiplication come before addition and subtraction
  - Put parentheses around everything else

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Unsigned Data Types

- We've seen

```
int a = -1;
long b = -1;
```

- Unsigned data types are always positive

```
unsigned a = 1;
unsigned long b = 1;
```

- Unsigned literals

`1U`	1 as unsigned constant
`1LU`	1 as unsigned long constant

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Limits for Computer Integers

- Limits for unsigned integers (`unsigned`, not `unsigned long`)
  - UINT_MIN = 0
  - UINT_MAX = $2^w - 1$
- Limits for signed integers (`int`, not `long int`)
  - INT_MIN = -2^{w-1}
  - INT_MAX = $2^{w-1} - 1$
  - Note: the textbook **incorrectly** implies that `INT_MIN == -INT_MAX` (section B11)
- *w* is machine dependent
  - *w* = 32 on the GRACE machines
  - `UINT_MAX`, `INT_MIN` and `INT_MAX` are defined as constants in `limits.h`

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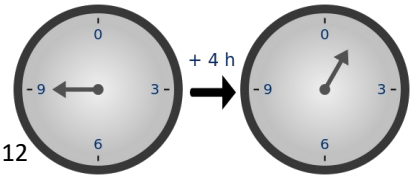
Integer Overflow

- What happens when you add 1 to UINT_MAX?
  - The mathematical value $(2^w - 1) + 1 = 2^w$ cannot be stored in an `unsigned` variable
  - The result of the operation is 0
- Intuition: unsigned numbers **wrap around**
  - Think of a 12h clock
  - 11 o'clock + 1h = 0
  - We count time **modulo 12h**
  - This means that the time displayed is the **remainder** from a division by 12



Source: Wikipedia

- **Unsigned operations are done modulo 2^w**

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Unsigned Integer Addition

- Mathematical addition: **infinite** precision
  - s = $u + v$
- `unsigned` addition: operations done **modulo 2^w**
  - `s` = $(u + v) \mod 2^w$
  - Example: UINT_MAX + 1 = $2^w \mod 2^w = 0$ (**overflow**)
- Multiplication can overflow in similar manner
  - Same for addition and multiplication of signed integers

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Properties of Signed Integers – Examples

- You can represent more negative than positive numbers
  - Positive range: 1 .. $(2^{w-1} - 1)$
  - Negative range: -1 .. -2^{w-1}
- Signed integers can overflow as well
  - INT_MAX + 1 = INT_MIN
    - **Adding 2 positive numbers may produce a negative number!**
  - INT_MIN – 1 = INT_MAX
    - **Adding 2 negative numbers may produce a positive number!**
  - INT_MIN = -INT_MIN
    - INT_MIN is its own inverse

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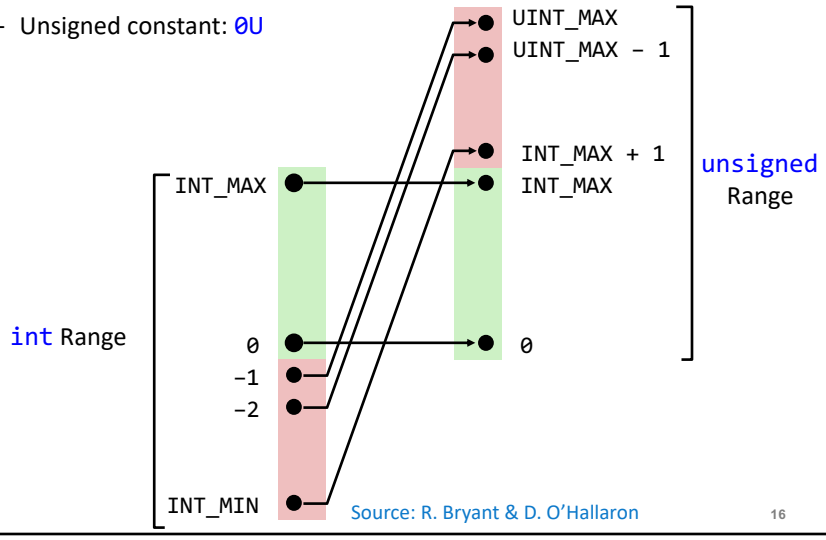
Conversion Between Signed and Unsigned

- Type conversion `int` → `unsigned` visualized
  - Signed int constant: `0`
  - Unsigned constant: `0U`



Source: R. Bryant & D. O'Hallaron

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Mathematical Properties of Integer Arithmetic

- Closed under addition and multiplication
  - Result of signed/unsigned operation is also a signed/unsigned integer
- Commutative
- Associative
- 0 is additive identity; 1 is multiplicative identity
- Multiplication distributes over addition
  - a * (b + c) = a*b + a*c

- **Does not obey the ordering properties** of math integers

  $u > 0$ $\quad \not\Rightarrow \quad u + v > v$

  $u > 0, v > 0$ $\quad \not\Rightarrow \quad u \cdot v > 0$

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Properties of Floating Point Numbers

- As many negative as positive numbers

- Special values (constants for some of these defined in `float.h`)
  - Max floating point number ⇒ operations may **overflow**
  - Min floating point > 0 ⇒ operations may **underflow**
  - Smallest ε such that 1.0 + ε ≠ 1.0 ⇒ operation results may be **rounded**
  - +Inf, -Inf, NaN (not a number)

- **Avoid testing the equality** of values resulting from floating point operations

```
if (FLT_MAX == (FLT_MAX+1)) {…}     condition is true
if (cos(M_PI / 2) != 0.0) {…}       condition is true
```

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Mathematical Properties of Floating Point Arithmetic

- Closed under addition and multiplication
  - But may generate infinity or NaN
- Commutative
- **Not associative**
  - (a + b) + c ≠ a + (b + c)
  - (a * b) * c ≠ a * (b * c)
  - Possibility of overflow, inexactness of rounding
- Multiplication **does not distribute** over addition
  - a * (b + c) ≠ a * b + a * c
  - Possibility of overflow, inexactness of rounding
- Monotonicity
  - a ≥ b ⇒ a+c ≥ b+c
  - a ≥ b & c ≥ 0 ⇒ a * c ≥ b *c
  - **Exceptions**: ±Inf and NaN

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Review of Lecture

- What did we learn?

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Next Steps

- Next lecture
  - Data types and type conversion
- Assignments for this week
  - Read **K&R Chapters 2.2, 2.9, 3.3, 6.1, B5, B6**
    - Note: some of these chapters refer to strings (e.g. `char s[]`), which we'll cover later
    - For now, think of `s[i]` as a character variable
  - Weekly challenge: **dec2bin.c**
  - Homework: `lab05.pdf` (on http://ter.ps/enee140), due on Friday at 11:59 pm
  - No quiz next week

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